Mark Scheme (Results)

October 2017

Pearson Edexcel International A-Level
In Core Mathematics C12 (WMA01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\left|pq\right|=\left|c\right|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\left|pq\right|=\left|c\right|\ \text{and}\ \left|mn\right|=\left|a\right|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n\rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n\rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	The line l_1 has equation $8x+2y-15=0$	
(a)	Gradient is -4	B1 **[1]**
(b)	Gradient of parallel line is equal to their previous gradient Equation is $y-16="-4"\left(x-\left(-\tfrac{3}{4}\right)\right)$ So $y=-4x+13$	M1 M1 A1 **[3]** **(4 marks)**

(a)

B1 Gradient, m, $\frac{\mathrm{d}y}{\mathrm{d}x}$ given as -4 FINAL ANSWER

Do not allow $-\frac{8}{2}$ or $-\frac{4}{1}$ or $-4\rightarrow\frac{1}{4}$ in part (a). Do not allow if left as $y=-4x+..$

(b)

M1 Gradient of lines are the same. This may be implied by sight of their $'-4'$ in a gradient equation. For example you may see candidates state $y='-4'x+..$ in (a) and then write $y='-4'x+c$ in (b)

M1 For an attempt to find an equation of a line using $\left(-\frac{3}{4},16\right)$ and a numerical gradient (which may be different to the gradient used in part (a)). For example they may try to find a normal! Condone a sign error on one of the brackets. If the form $y=mx+c$ is used they must proceed as far as finding c.

A1 cao $y=-4x+13$ Allow $m=-4, c=13$

Question Number	Scheme	Marks
2.(a)	$(0,3)$	B1
(b)	$(2,-3)$	B1
(c)	$(2,1.5)$ oe	B1
(d)	$(2,-1)$	B1 **[4]** **(4 marks)**

Condone the omission of the brackets. Eg Condone $0,3$ for $(0,3)$

Allow $x=...y=...$

If options are given, Attempt one $=(0,3)$, Attempt two = $(2,5)$, Award B0.

If there is no labelling mark (a) as the first one seen, (b) as the second one seen etc unless it is obvious.

Question Number	Scheme	Marks
3.(a)	$\frac{x^3+4}{2x^2}=\frac{x^3}{2x^2}+\frac{4}{2x^2}=\frac{1}{2}x+2x^{-2}$	M1A1A1 **[3]**
(b)	$\int\frac{x^3+4}{2x^2}\,\mathrm{d}x=\int\frac{1}{2}x+2x^{-2}\,\mathrm{d}x=\frac{1}{4}x^2-2x^{-1}+c$	M1A1A1 **[3]** **(6 marks)**

(a)

M1 For an attempt to divide by $2x^2$. It may be implied if either index or either coefficient is correct.

A1 One correct term. Either $\frac{1}{2}x$ or $+2x^{-2}$. Allow $\frac{1}{2}x^1=0.5x$ or, for this mark only, $+2x^{-2}=+\frac{2}{x^2}$

A1 $\frac{1}{2}x+2x^{-2}$ or $0.5x+2x^{-2}$ Accept $x^1=x$ A final answer of $\frac{1}{2}x+\frac{2}{x^2}$ is M1 A1 A0

(b)

M1 Raises any of the indices by one for their Ax^p+Bx^q

A1 One term both correct and simplified. Accept either $\frac{1}{4}x^2\,/\,0.25x^2$ or $-2x^{-1}\,/-\frac{2}{x}\,/-\frac{2}{x^1}$

A1 $\frac{1}{4}x^2-2x^{-1}+c$ including the +c. Accept equivalents such as $0.25x^2-\frac{2}{x^1}+c$ or $\frac{x^3-8}{4x}+c$

Do not accept expressions like $\frac{1}{4}x^2+-2x^{-1}+c$

Question Number	Scheme	Marks
4.(a)	Attempts Area $= \frac{1}{2}ab\sin C \Rightarrow 24\sqrt{3} = \frac{1}{2}3x \times x\sin 60°$	M1
	Uses $\sin 60° = \frac{\sqrt{3}}{2}$ oe $\Rightarrow x^2 = 32 \Rightarrow x = 4\sqrt{2}$	dM1A1*
		[3]
(b)	Uses $BC^2 = \left(12\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2 - 2\left(12\sqrt{2}\right)\left(4\sqrt{2}\right)\cos 60°$	M1
	$\Rightarrow BC^2 = 224 \Rightarrow BC = 4\sqrt{14}$	A1,A1
		[3] **(6 marks)**

(a)

M1 Attempts to use Area $= \frac{1}{2}ab\sin C$ Score for sight of $24\sqrt{3} = \frac{1}{2}3x \times x\sin 60°$

dM1 Either using $24\sqrt{3} = \frac{1}{2}3x \times x\sin 60°$ with $\sin 60° = \frac{\sqrt{3}}{2}$ (which may be implied) to reach a form $x^2 = k$

So sight of $x^2 = \frac{16\sqrt{3}}{\sin 60°}$ oe $\Rightarrow x = 4\sqrt{2}$ would imply $\sin 60° = \frac{\sqrt{3}}{2}$ and $x^2 = k$

Or sight of a correct simplified intermediate line followed by the correct answer.

Eg. $24\sqrt{3} = \frac{1}{2}3x \times x\sin 60° \Rightarrow 3x^2 = 96 \Rightarrow x = 4\sqrt{2}$

It cannot be awarded for $24\sqrt{3} = \frac{1}{2}3x \times x \times \frac{\sqrt{3}}{2} \Rightarrow x = 4\sqrt{2}$

A1* This is a show that and you must see $x = 4\sqrt{2}$ following $x^2 = 32$ OR $x^2 = 16 \times 2$ or $x = \sqrt{32}$ for the A1* to be scored

If you see a candidate start $41.57 = \frac{1}{2}3x \times x \times 0.866 \Rightarrow x^2 = 32 \Rightarrow x = 4\sqrt{2}$ award M1, dM1, A0

Alternatively candidate can assume that $x = 4\sqrt{2}$ and attempt $\frac{1}{2}4\sqrt{2} \times 12\sqrt{2}\sin 60°$ for M1, $\frac{1}{2}4\sqrt{2} \times 12\sqrt{2} \times \frac{\sqrt{3}}{2} = 24\sqrt{2}$ for dM1 and make a statement for A1*

(b)

M1 Uses the cosine rule $BC^2 = \left(4\sqrt{2}\right)^2 + \left(12\sqrt{2}\right)^2 - 2\left(4\sqrt{2}\right)\left(12\sqrt{2}\right)\cos 60°$ Condone missing brackets

Can be scored for $BC^2 = (3x)^2 + (x)^2 - 2(3x)(x)\cos 60°$ It can be awarded for an attempt with their x

Also accept the form $\cos 60° = \frac{\left(12\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2 - BC^2}{2\left(12\sqrt{2}\right)\left(4\sqrt{2}\right)}$

A1 $BC^2 = 224$ May be implied by $BC = \sqrt{224}$ or $4\sqrt{14}$

A1 $BC = 4\sqrt{14}$

If you see a candidate start $BC^2 = (5.66)^2 + (16.97)^2 - 2(5.66)(16.97)\cos 60° \Rightarrow BC = 4\sqrt{14}$
award M1, A1, A0

Question Number	Scheme	Marks
5.(a)	$y = 27x^{0.5} - 2x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$	M1A1A1
		[3]
(b)	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1
	$\frac{27}{2}x^{-0.5} - 4x = 0 \Rightarrow x^{1.5} = \frac{27}{8} \Rightarrow x = \frac{9}{4}$	dM1,A1
	$x = \frac{9}{4} \Rightarrow y = \frac{243}{8}$	dM1A1
		[5] **(8 marks)**

(a)

M1 Uses $x^n \to x^{n-1}$ at least once. So sight of either index $x^{-0.5}$ / $x^{-\frac{1}{2}}$ or $x = x^1$

A1 Either term correct (may be unsimplified). Eg. $2 \times 2x^1$ is acceptable. The indices must be tidied up however so don't allow $2 \times 2x^{2-1}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ or exact equivalent such as $\frac{\mathrm{d}y}{\mathrm{d}x} = 13.5 \times \frac{1}{\sqrt{x}} - 4x$.
It must be all tidied up for this mark so do not allow $2 \times 2x$

(b)

M1 States or sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ This may be implied by subsequent working.

dM1 Dependent upon the previous M and correct indices in (a). It is awarded for correct index work leading to $x^{\pm 1.5} = k$ Also allow squaring $27x^{-0.5} = 8x \Rightarrow \frac{27^2}{x} = 64x^2 \Rightarrow x^3 =$

A1 $x = \frac{9}{4}$ or exact equivalent. A correct answer following a correct derivative can imply the previous mark provided you have not seen incorrect work.

dM1 Dependent upon the first M1 in (b). For substituting their value of x into y to find the maximum point. There is no need to check this with a calculator. (y appearing from an x found from $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ is fine.)

A1 $y = \frac{243}{8}$ or exact equivalent (30.375) . You do not need to see the coordinates for this award.
Ignore any other solutions outside the range. If extra solutions are given within the range withhold only this final mark.

Note: This question requires differentiation in (a) and minimal working in (b). A correct answer without any differentiation will not score any marks.

Allow (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ (b) $0 = \frac{27}{2}x^{-0.5} - 4x \Rightarrow x = \frac{9}{4}, y = \frac{243}{8}$ for all marks

Whereas (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ (b) $x = \frac{9}{4}, y = \frac{243}{8}$ scores (a) 3 (b) 0 marks

Question Number	Scheme	Marks
6. (a)	Uses $1000 = 600 + 80(N-1) \Rightarrow N = 6$	M1,A1 **[2]**
(b)	Uses $\frac{15}{2}\left(2\times600+(15-1)\times80\right)=(£)17400$	M1 A1 **[2]**
(c)	Total for Saima = $\frac{15}{2}\left(2A+14A\right)=\left(120A\right)$ Sets $120A = 17400 \Rightarrow A = 145$	B1 M1A1 **[3]** **(7 marks)**

(a)

M1 Attempts to use the formula $u_n = a+(n-1)d$ to find the value of 'n'.

Evidence would be $1000 = 600 + 80(N-1)$

Alternatively attempts $\frac{1000-600}{80}+1$ or repeated addition of £80 onto £600 until £1000 is reached

A1 $N = 6$ or accept the 6th year (or similar). The answer alone would score both marks.

(b)

M1 Uses a correct sum formula $S = \frac{n}{2}\left(2a+(n-1)d\right)$ with $n = 15, a = 600, d = 80$

Alternatively uses $S = \frac{n}{2}\left(a+l\right)$ with $n = 15,\ a = 600,\ l = 600+14\times80$ or 1720

Accept the sum of 15 terms starting 600 + 680+ 760+ 840+....

A1 cao (£)17400

(c)

B1 Finds the sum for Saima.

Accept unsimplified forms such as $\frac{15}{2}\left(2A+14A\right)$ or $\frac{15}{2}\left(A+15A\right)$ or the simplified answer of $120A$

Remember to isw following a correct answer

M1 Sets their $120A$ equal to their answer to (b) and proceeds to find a value for A.

They must be attempting to calculate sums rather than terms to score this mark.

Condone slips on the sum of an AP formula and award for a valid attempt from GP formula.

A1 cao $A = 145$

Question Number	Scheme	Marks
7.	$\mathrm{g}(x)=2x^3+ax^2-18x-8$	
(a)	$\mathrm{g}(\pm 2)=0 \Rightarrow 2\left(\pm 2\right)^3+a\left(\pm 2\right)^2+18\left(\pm 2\right)-8=0$	M1
	$\Rightarrow 4a=-12 \quad \Rightarrow a=-3$	A1*
		[2]
(b)	$\mathrm{g}(x)=2x^3-3x^2-18x-8=(x+2)(2x^2-7x-4)$	M1 A1
	$=(x+2)(2x+1)(x-4)$	M1A1
		[4]
(c)	$\sin\theta=-\frac{1}{2}$ only	B1ft
	$\theta=\frac{7}{6}\pi,\frac{11}{6}\pi$	M1A1
		[3]
		(9 marks)

(a)

M1 Attempts $\mathrm{g}(\pm 2)=0$ This can be implied by subsequent working

Alternatively divides by $(x+2)$ and sets the remainder equal to 0

For division look for a minimum of

$$\begin{array}{r}2x^2+(a...)x+(...a).....\\ x+2\overline{)2x^3+ax^2-18x-8}\end{array}$$

$$\begin{array}{r}\overline{\qquad\qquad}\\(...a)+....\end{array}$$

followed by the remainder (involving a) set equal to 0

A1* $a=-3$ or equivalent following a correct linear equation in $4a$ that is readily solvable.

(As a rule accept $4a=-12$ or similar such as $4a=8-36+16$ or $4a+12=0$ I am classing as readily solvable)

Note that this is a given answer and so the candidate **must** proceed from $-16+4a+36-8=0$ oe to score this mark. Expect to see (as a bare minimum) one calculation/process that makes it more solvable.

So $-16+4a+36-8=0 \Rightarrow 4a+20-8=0$ could be seen as the bare minimum.

(b)

M1 Attempts to divide $g(x)$ by $(x+2)$ to produce the quadratic factor.

For division look for the first two terms

$$\begin{array}{r} 2x^2 - 7x \;+ .... \\ x+2\overline{\smash{)}2x^3-3x^2-18x-8} \\ \underline{2x^3+4x^2} \\ -7x^2 \end{array}$$

For factorisation/ inspection look for the first and last terms $2x^3-3x^2-18x-8=(x+2)(2x^2.....x-4)$.

A1 The correct quadratic factor $(2x^2-7x-4)$

M1 Attempts to factorise the quadratic factor using usual rules. This must appear in part (b)

A1 $g(x)=(x+2)(2x+1)(x-4)$. Accept $g(x)=2(x+2)\left(x+\frac{1}{2}\right)(x-4)$

All factors must appear on the same line.

Note: the question asks the candidate to use algebra to factorise $g(x)$

Candidates who write down $g(x)=0 \Rightarrow x=-2,-\frac{1}{2},4 \Rightarrow g(x)=(x+2)\left(x+\frac{1}{2}\right)(x-4)$ score 0000

Candidates who write down $g(x)=0 \Rightarrow x=-2,-\frac{1}{2},4 \Rightarrow g(x)=(x+2)(2x+1)(x-4)$ oe score 1000

(c)

B1ft States or implies that $\sin\theta=-\frac{1}{2}$ only. Follow through on all roots $-1\leqslant \sin\theta \leqslant 1$

As long as they don't find values from $\sin\theta=4$ or $\sin\theta=-2$ that implies they have "chosen" $\sin\theta=-\frac{1}{2}$

M1 Uses a correct method to solve an equation of the form $\sin\theta=k, -1\leqslant k\leqslant 1$ by ‘arcsin’

You may need to check this using a calculator.

This may be implied by $\sin\theta=-\frac{1}{2} \Rightarrow \theta=-30°$

A1 $\theta=\frac{7}{6}\pi, \frac{11}{6}\pi$ or exact equivalent only.......within the given range. Ignore answers outside this range.

Condone $1.1\dot{6}$ for $\frac{7}{6}$ and $1.8\dot{3}$ for $\frac{11}{6}$

Question Number	Scheme	Marks
8.(a)	$r\theta = 6$ and $\frac{1}{2}r^2\theta = 20$	B1 B1 **[2]**
(b)	Substitute $r\theta = 6$ into $\frac{1}{2}r^2\theta = 20 \Rightarrow \frac{1}{2}\times 6r = 20$	M1
	$\Rightarrow r = \frac{20}{3}$	A1
	Substitutes $r = \frac{20}{3}$ in $r\theta = 6 \Rightarrow \theta = \frac{9}{10}$	dM1A1
		[4] **(6 marks)**

This may be marked as one complete question. Eg they may just give the equations $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in (a)
Don't penalise this sort of error.

(a)

B1 Either $r\theta = 6$ or $\frac{1}{2}r^2\theta = 20$ (or exact equivalents)

Allow $\frac{\theta}{2\pi}\times 2\pi r = 6$ or $\frac{\theta}{2\pi}\times \pi r^2 = 20$ but not $\frac{\theta}{360}\times 2\pi r = 6$ or $\frac{\theta}{360}\times \pi r^2 = 20$

B1 Both $r\theta = 6$ and $\frac{1}{2}r^2\theta = 20$ (or exact equivalents)

Allow $\frac{\theta}{2\pi}\times 2\pi r = 6$ and $\frac{\theta}{2\pi}\times \pi r^2 = 20$ but not $\frac{\theta}{360}\times 2\pi r = 6$ and $\frac{\theta}{360}\times \pi r^2 = 20$

(b)

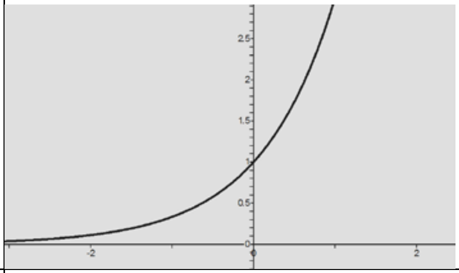
M1 Combines two equations in r and θ producing an equation in one unknown.

A1 $r = \frac{20}{3}$ or $\theta = \frac{9}{10}$ or exact equivalents.

You may just see answers following correct equations. This is fine for all the marks

dM1 This is dependent upon having started with two equations with correct expressions in r and θ
Look for $..r\theta = ...$ and $..r^2\theta = ...$.
It is awarded for correctly substituting their value of r or θ into one of the equations to find the second unknown.

A1 $r = \frac{20}{3}$ and $\theta = \frac{9}{10}$ or exact equivalents. Condone $6.\dot{6}$ for $\frac{20}{3}$ Do not allow 6.67

Question	Scheme	Marks
9 (a)	 Shape **or** y intercept at 1	B1
	Fully correct shape and intercept	B1
		[2]
(b)	State h = 2, or use of $\frac{1}{2}\times 2$;	B1
	$\underline{\left\{0.0625+16+2(\ 0.25+1+4\ )\right\}}$ For structure of $\{................\}$;	M1A1
	$\frac{1}{2}\times 2\times\underline{\{26.5625\}} = awrt\ 26.56$ Exact answer = $^{425}\!\!/_{16}$	A1cao
		[4]
(c)(i)	$4\times(b) = awrt\,106$ Exact answer = $^{425}\!\!/_{4}$	M1A1ft
(ii)	$24+(b) = awrt\ 50.6$ Exact answer = $^{809}\!\!/_{16}$	M1A1ft
		[4] **(10 marks)**

(a)

B1 Score for either

- a correct shape for the curve. It must lie only in quadrants 1 and 2 and have a positive and increasing gradient from left to right. The gradient must be approximately 0 at the left hand end. Condone the curve appearing to be a straight line on the rhs. See Practice/Qualification items for clarification. Do not be concerned if it does not appear to be asymptotic to the *x*-axis at the LHS
- intercept at (0,1). Allow 1 being marked on the *y* - axis. Condone (1,0) on the correct axis.

B1 Fully correct. As a guide the gradient of the curve must appear to be 0 at the lh end and it must reach a level that is more than half way below the level of the intercept at (0,1). Allow $x=0, y=1$ in the text, it does not need to be on the sketch. Do not condone (1,0) even on the correct axis for this mark.

(b)

B1 For using a strip width of 2. This may appear in a trapezium rule as $\frac{1}{2}\times 2$ or 1 or equivalent

M1 Scored for the correct $\{......\}$ outer bracket structure. It needs to contain first *y* value plus last *y* value and the inner bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from inner bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are *x* values instead of *y* values

A1 For the correct bracket $\{......\}$

A1 For awrt 26.56. Accept 425/16

NB: Separate trapezia may be used: B1 for *h* =1, M1 for 1/2 *h*(*a* + *b*) used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Note: As *h* =1 the expression $1\times(16+0.0625)+2(0.25+1+4)$ will scores B1 M1 A1 with awrt 26.56 scoring the final A1.

(c)(i)

M1 For an attempt at finding $4\times(b)$. Also allow repeating the trapezium rule with each value $\times 4$

A1ft For either awrt 106 or ft on the answer to $4\times(b)$ You may see $425/4$ following 425/16 in (b)

(c)(ii)

M1 For an attempt at $24+(b)$ or $[3x]_{-4}^{4}+(b)$ Also allow repeating the trapezium rule with each value $+3$

A1ft For either awrt 50.6 or ft on the answer to 24+(b) You may see 809/16

Question Number	Scheme	Marks
10.(a)	$p=13, q=13$	B1 B1 **[2]**
(b)	Gradient *AD*/ *AC* / *DC*= $\frac{5-(-3)}{10-7}=\left(\frac{8}{3}\right)$	M1
	Gradient $DE = -\frac{3}{8}$	M1, A1
	Equation of l is $(y-5)="-\frac{3}{8}"(x-10)\Rightarrow 3x+8y=70$	M1A1 **[5]**
(c)	Sub $x=7$ into $3x+8y=70\Rightarrow y=\frac{49}{8}$. Hence $C=\left(7,\frac{49}{8}\right)$	M1A1 **[2]** **(9 marks)**

(a)

B1 For either $p=13$ or $q=13$. Score within a coordinate $(13,...)$ or $(...,13)$ Just 13 scores B1B0

B1 For both $p=13$ and $q=13$. Allow $(13,13)$ for both marks.

(b)

M1 For an attempt at the gradient of *AD* or *AC* using their coordinates for *C*

Look for an attempt at $\frac{\Delta y}{\Delta x}$ There must be an attempt to subtract on both the numerator and the denominator. It can be implied by their attempt to find the equation of line *AC*

M1 For an attempt at using $m_2=-\frac{1}{m_1}$ or equivalent to find the gradient of the perpendicular m_2

A1 Gradient of *DE* is $-\frac{3}{8}$ or equivalent

M1 It is for the method of finding a line passing though (10, 5) with a changed gradient. Eg $\frac{8}{3}\rightarrow\frac{3}{8}$

Look for $(y-5)=\text{changed } m_1(x-10)$ Both brackets must be correct

Alternatively uses the form $y=mx+c$ AND proceeds as far as c = ..

A1 $3x+8y=70$ or exact equivalent. Accept $\pm A(3x+8y=70)$ where $A\in\mathbb{N}$

(c)

M1 Substitutes $x=7$ in their $3x+8y=70\Rightarrow y=...$

A1 $C=\left(7,\frac{49}{8}\right)$ or exact equivalent. Allow this mark when x and y are written separately.

Do not allow this A1 if other answers follow $x=7$ $y=\frac{49}{8}$

Question Number	Scheme	Marks
11.(a)	$\left(3+ax\right)^5 = 3^5 + \begin{pmatrix}5\\1\end{pmatrix}3^4.\left(ax\right)+\begin{pmatrix}5\\2\end{pmatrix}3^3.\left(ax\right)^2+...$	M1
	$= 243, +405ax + 270a^2x^2 + ...$	B1, A1, A1
		[4]
(b)	$\text{f}(x) = (a-x)(3+ax)^5 = (a-x)(243+405ax+270a^2x^2+...)$	
	$-243+405a^2 = 0 \Rightarrow a^2 = \dfrac{243}{405} \Rightarrow a = \sqrt{\dfrac{3}{5}}$ or equivalent	M1,dM1A1
		[3] **(7 marks)**

(a)

M1 This method mark is awarded for an attempt at a Binomial expansion to get the second and/or third term – it requires a correct binomial coefficient combined with correct power of 3 and the correct power of *x*. Ignore bracketing errors. Accept any notation for ${}^5C_1$, ${}^5C_2$, e.g. as on scheme or 5, and 10 from Pascal's triangle. This mark may be given if no working is shown, if either or both of the terms including *x* is correct.

An alternative is $\left(3+ax\right)^5 = 3^5\left\{1+\dfrac{ax}{3}\right\}^5 = 3^5\left\{1+5\times\dfrac{ax}{3}+\dfrac{5\times 4}{2(!)}\times\left(\dfrac{ax}{3}\right)^2\right.$

In this method it is scored for the correct attempt at a binomial expansion to get the second and/or third term in the bracket of $3^n\left\{1+5\times\dfrac{ax}{3}+\dfrac{5\times 4}{2(!)}\times\left(\dfrac{ax}{3}\right)^2 ......\right\}$

Score for binomial coefficient with the correct power of $\left(\dfrac{x}{3}\right)$ Eg. $5\times\dfrac{..x}{3}$ or $10\times\left(\dfrac{..x}{3}\right)^2$

B1 Must be simplified to 243 (writing just 3^5 is B0).

A1 cao and is for one correct from 405*ax*, and $270a^2x^2$ Also allow $270(ax)^2$ with the bracket

A1 cao and is for both of $405a\,x$, and $270a^2x^2$.

Allow $270(ax)^2$ with the bracket correct (ignore extra terms). Allow listing for all marks

It is possible to score 1011 in (a)

There are a minority of students who attempt this in (a)

$\text{f}(x) = (a-x)(3+ax)^5 = (a-x)(243+405ax+270a^2x^2+...)$ and go on to expand this.

They can have all the marks in part (a)

(b)

M1 Attempt to set the coefficient of *x* in the expansion of $(a-x)(3+ax)^5$ equal to 0

$(a-x)(3+ax)^5 = (a-x)\left(P+Qax+Ra^2x^2+...\right) = aP+\left(a^2Q-P\right)x+...$

For this to be scored you must see an equation of the form $\pm P \pm Qa^2 = 0$ You are condoning slips/ sign errors

dM1 For $\pm P \pm Qa^2 = 0 \Rightarrow a = ...$ using a correct method. This cannot be scored for an attempt at sq rooting a negative number

A1 $a = \sqrt{\dfrac{3}{5}}$ or exact equivalent such as $a = \dfrac{\sqrt{15}}{5}$ You may ignore any reference to $a = -\sqrt{\dfrac{3}{5}}$

Question	Scheme	Marks
12. (i)	$3\sin(\theta+30^\circ)=2\cos(\theta+30^\circ)\Rightarrow\tan(\theta+30^\circ)=\frac{2}{3}$	M1
	$\Rightarrow\theta+30^\circ=\arctan\left(\frac{2}{3}\right)=33.69^\circ,213.69^\circ\Rightarrow\theta=..$	dM1
	$\Rightarrow\theta=3.69^\circ,183.69^\circ$	A1, A1 **[4]**
Alt (i)	$3\sin(\theta+30^\circ)=2\cos(\theta+30^\circ)\Rightarrow3(\sin\theta\cos30^\circ+\cos\theta\sin30^\circ)=2(\cos\theta\cos30^\circ-\sin\theta\sin30^\circ)$	
	$\div\cos\theta\Rightarrow3\tan\theta\cos30^\circ+3\sin30^\circ=2\cos30^\circ-2\tan\theta\sin30^\circ$	M1
	$\Rightarrow\tan\theta=\frac{2\cos30^\circ-3\sin30^\circ}{3\cos30^\circ+2\sin30^\circ}\left(=\text{awrt }0.0645\right)$	dM1
	$\Rightarrow\theta=3.69^\circ,183.69^\circ$	A1 A1
		[4]
(ii)(a)	$\frac{\cos^2x+2\sin^2x}{1-\sin^2x}=5\Rightarrow\frac{\cos^2x+2\sin^2x}{\cos^2x}=5$	M1
	$\Rightarrow1+2\tan^2x=5$	M1
	$\Rightarrow\tan^2x=2$	A1
(ii)(b)	$\tan^2x=2\Rightarrow\tan x=\pm\sqrt{2}$	M1
	$\Rightarrow x=0.955,2.186,4.097,5.328$	M1 A1,A1
		[7]
		(11 marks)

(i)

M1 For stating that $\tan(\theta+30^\circ)=k,\quad k\neq0$ Allow even where the candidate writes $\tan(\theta+30^\circ)=\frac{3}{2}$

dM1 For taking 'arctan' subtracting 30 and proceeding to $\theta=..$ Do not allow mixed units

For $\tan(\theta+30^\circ)=\frac{3}{2}$ it is scored when they reach $\theta=26.3^\circ$

A1 $\theta=3.69^\circ$ or 183.69°

A1 $\theta=3.69^\circ$ and 183.69° only in the range $0\rightarrow360$

(ii)(a)

M1 For use of $1-\sin^2x=\cos^2x$ or equivalent.

This may be scored either by setting $\frac{\cos^2x+2\sin^2x}{1-\sin^2x}=\frac{\cos^2x+2\sin^2x}{\cos^2x}$ or $\frac{\cos^2x+2\sin^2x}{1-\sin^2x}=\frac{1+\sin^2x}{1-\sin^2x}$

M1 For dividing **both terms** by $\cos^2x$ and using $\frac{\sin^2x}{\cos^2x}=\tan^2x$ leading to $\tan^2x=k$

In the alternative $\sin^2x=c\Rightarrow\tan^2x=k$ can be done on a calculator

A1 $\tan^2x=2$

(ii)(b)

M1 For taking the square root and stating that $\tan x=\sqrt{k}$ (or $\tan x=-\sqrt{k}$). Accept decimals here. One correct angle would imply this. Allow a solution from $\sin^2x=c$

M1 For taking arctan and finding two of the 4 angles for their $\tan x=\sqrt{k}$ (or $\tan x=-\sqrt{k}$) (Alt for taking arcsin or arcos and finding 2 angles)

Condone slips here. For example, $\tan^2x=2\Rightarrow\tan x=\pm2$ can score M0 M1 if two angles are found.

BUT for example $\tan^2x=2\Rightarrow\tan x=2$ leading to two answers scores M0 M0

A1 Two of awrt $x=0.96,2.19,4.10,5.33$.

Accept degrees here ie accept two of $54.7^\circ,125.3^\circ,234.7^\circ,305.3^\circ$

A1 All four angles in radians (and no extra's within the range) awrt $x=0.955,2.186,4.097,5.328$

Question Number	Scheme	Marks
13 (a)(i)	$(3,-4)$	B1
(a)(ii)	$\sqrt{30}$	B1
		[2]
(b)	Attempts $(6-3)^2+(k+4)^2, < 30$	M1,M1
	$k^2+8k-5<0$	A1*
		[3]
(c)	Solves quadratic by formula or completion of square to give $k =$	M1
	$k=-4\pm\sqrt{21}$	A1
	Chooses region between two values and deduces $-4-\sqrt{21}<k<-4+\sqrt{21}$	M1
		A1**cao**
		[4]
		(9 marks

(a)(i)(ii)

B1 $(3,-4)$ Accept as $x =\ , y =$ or even without the brackets

B1 $\sqrt{30}$ Do not accept decimals here but remember to isw

(b) This is scored M1 A1 A1 on e -pen. We are marking it M1 M1 A1

M1 Attempts to find the length or length^2 from P $(6,k)$, to the centre of C $(3,-4)$ following through on their C. Look for, using a correct C, either $(6-'3')^2+(k+'4')^2$ or $\sqrt{(6-'3')^2+(k+'4')^2}$

Another way is to substitute $(6,k)$ into $(x-3)^2+(y+4)^2=30$ but it is very difficult to score either of the other two marks using this method.

M1 Forms an inequality by using the length from P to the centre of C < the radius of C $(6-3)^2+(k+4)^2<30$. In almost all cases I would expect to see < 30 before < 0

Using the alternative method, they would also need the line $(6-3)^2+(k+4)^2<30$. (As if the point lies on another circle, the radius/distance would need to be smaller than 30)

A1* $k^2+8k-5<0$

This is a given answer and you must check that all aspects are correct. **In most cases you should expect to see an intermediate line (with < 30) before the final answer appear with < 0.**

(c)

M1 Solves the equation $k^2+8k-5=0$ by formula or completing the square.

Factorisation to integer roots is not a suitable method in this case and scores M0.

The answers could just appear from a graphical calculator. Accept decimals for the M's only

A1 Accept $k=-4\pm\sqrt{21}$ or exact equivalent $k=\dfrac{-8\pm\sqrt{84}}{2}$

Do not accept decimal equivalents $k=-8.58,(+)0.58$ 2dp for this mark

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation.

A1 cao $-4-\sqrt{21}<k<-4+\sqrt{21}$ Accept equivalents such as $\left(-4-\sqrt{21},-4+\sqrt{21}\right)$, $k>-4-\sqrt{21}$ and $k<-4+\sqrt{21}$, even $k>-4-\sqrt{21}$, $k<-4+\sqrt{21}$

Accept for 3 out of 4 $\left[-4-\sqrt{21},-4+\sqrt{21}\right]$, $k>-4-\sqrt{21}$ or $k<-4+\sqrt{21}$, $-4-\sqrt{21}\leqslant k\leqslant -4+\sqrt{21}$

Do not accept $-4-\sqrt{21}<x<-4+\sqrt{21}$ for this final mark

Question Number	Scheme	Marks
14 (a)	$u_6 = 8000\times(0.85)^5 = 3549.6 \approx 3550$	M1, A1 **[2]**
(b)	States $\lvert r \rvert < 1$ **or** 0.85 < 1 **and makes no reference to terms**	B1 **[1]**
(c)	$S_\infty = \frac{a}{1-r} = \frac{8000}{1-0.85} = \text{awrt } 53333 \quad 53334 \quad \frac{160\,000}{3}$	M1A1 **[2]**
(d)	Uses $S_N = \frac{8000(1-0.85^N)}{1-0.85}$	M1
	$\frac{8000(1-0.85^N)}{1-0.85} = 40000 \Rightarrow 0.85^N = 0.25$	dM1 A1
	$\Rightarrow N = \frac{\log 0.25}{\log 0.85}(=8.53) \Rightarrow N = 9$	M1 A1
		[5] **[10 marks]**

(a)

M1 Attempts $u_6 = 8000\times(r)^5$ with $r = 0.85$ or 85% or $1-0.15$ or 1-15%

A1* Completes proof. States $u_6 = 8000\times(0.85)^5$ oe (see above) and shows answer is awrt 3549.6 or 3550

(b)

B1 States $\lvert r\rvert < 1$ or 0.85 < 1 **and makes no reference to terms**

Allow $r < 1$ $\quad -1 < r < 1$ **and makes no reference to terms**

Allow for an understanding of why S_∞ exists. Accept $0.85^n \to 0$ as $n \to \infty$ or $r^n \to 0$ as $n \to \infty$

Do not allow from an incorrect statement... if they give r = 0.15

Do not allow on an explanation that is based around terms.

Eg Do not allow $8000\times 0.85^{n-1} \to 0$ as $n \to \infty$

Do not allow as $r < 1$ $u_n \to 0$ and so a limit exists

Do not allow if they state 85% is less than 100%

If you feel that a candidate deserves this mark then please seek advice.

(c)

M1 Attempts $S_\infty = \frac{8000}{1-r}$ with $r = 0.85$ oe

A1 $\frac{8000}{1-0.85}$ with an answer of awrt 53333 or $53\,334$ or $\frac{160\,000}{3}$

(d)

M1 Uses $S_N = \dfrac{8000\left(1-r^N\right)}{1-r}$ with $r = 0.85$ oe and $S_N = 40\,000$

Condone for this mark $r = 0.15$ oe

dM1 Rearranges $\dfrac{8000\left(1-r^N\right)}{1-r} = 40000$ to $r^N = k$ with $r = 0.85$ or 0.15 oe

A1 $0.85^N = 0.25$

M1 Uses logs to solve an equation of the form $a^N = b \quad \left(a, b > 0\right)$ It must be a correct method and reach *N* = ...

If you see just the answer from $a^N = b$ look for accuracy of at least 1 dp

This can be scored starting from $40\,000 = 8000 \times \left('r'\right)^{N-1}$ but must proceed to *N* =..

A1 cso 9

Note: All marks in this part can be scored using inequalities as long as the final answer is 9. You may withhold the last mark if there are inconsistent inequality signs.

Accept trial and improvement. 1st M1 as above, 2nd M1 either sight of using *N*=8 or *N*=9, A1 correct,
3rd M1 for both *N*=8 and *N*=9, A1 correct answer.

FYI $S_8 = \dfrac{8000\left(1-0.85^8\right)}{1-0.85} = 38800$ AND $S_9 = \dfrac{8000\left(1-0.85^9\right)}{1-0.85} = 40980$

As the question does not have the magic phrase, we must also allow $\dfrac{8000\left(1-r^N\right)}{1-r} = 40000 \rightarrow N = 8.5 \Rightarrow N = 9$ for all marks. If the candidate just writes out line one and puts *N* = 9 we will allow special case 1 1 000

Question Number	Scheme		Marks
15. (a)(i)	18		B1
(ii)	3		B1
			[2]
(b)	$\frac{(x-3)^2(x+4)}{2}='18'$		M1
	$(x^2-6x+9)(x+4)=36$		
	$\Rightarrow x^3-2x^2-15x+36=36$		dM1
	$\Rightarrow x^3-2x^2-15x=0 \Rightarrow x^2-2x-15=0$		A1*
			[3]
(c)	$x=5$		B1
	$y=\frac{(5-3)^2(5+4)}{2} \Rightarrow (5,18)$		M1A1
			[3]
(d)	**Method 1**	**Method 2** OR	
	$\int\left(\frac{1}{2}x^3-x^2-\frac{15}{2}x+18\right)\mathrm{d}x=\frac{1}{8}x^4-\frac{1}{3}x^3-\frac{15}{4}x^2+18x$	$\int\left(-\frac{1}{2}x^3+x^2+\frac{15}{2}x\right)\mathrm{d}x=-\frac{1}{8}x^4+\frac{1}{3}x^3+\frac{15}{4}x^2$	M1A1
	Uses their 5 as the upper limit (and subtracts 0) to obtain an area	Uses their 5 as the upper limit (and subtracts 0) to obtain area	M1
	Area of rectangle = 90	Implied by correct answer $57\frac{7}{24}$	B1
	Use = Area of rectangle – Area beneath curve	Implied by subtraction in the integration	dM1
	$=90-32\frac{17}{24}=57\frac{7}{24}\quad\left(\frac{1375}{24}\right)$	$=57\frac{7}{24}\quad\left(\frac{1375}{24}\right)$	A1cso
			[6] **(14 marks)**

(a)(i)

B1 18 $P(0,18)$ or even P =18 is fine but do not allow $P(18,0)$

(a)(ii)

B1 3 $R(3,0)$ or even R =3 is fine but do not allow $R(0,3)$ or if they state 3 and -4

(b)

M1 Sets $\text{f}(x) = \text{their '18'}$ It can be implied by sight of $(x-3)^2(x+4) = 2\times$their 18

dM1 Attempts to multiply out $(x-3)^2(x+4)$ using a correct method. **Accept working for this expansion from elsewhere in the question. (It may be scribbled out which is fine BUT it must be seen**)
Expect to see $(x^2 \pm 6x \pm 9)(x+4)$ "multiplied" out to a cubic.

A1* Reaches the given answer of $x^2-2x-15=0$ following $x^3-2x^2-15x=0$ with no errors

(c)

B1 States $x=5$

M1 Attempts to find the y coordinate of Q by substituting their 5 into $\text{f}(x)$
Alternatively implies the y coordinate by using the same value as their answer to a(i)

A1 cao $(5,18)$ Allow written as $x=5, y=18$ It must be seen in part (c)

(d) Decide the method first:

……

Method one: Curve and line separate

M1 For integrating what they think is their $\text{f}(x)$ which must be cubic.
All powers must be raised by one for this to be scored.

A1 Correct $\frac{1}{8}x^4-\frac{1}{3}x^3-\frac{15}{4}x^2+18x$ which may be unsimplified

M1 Uses an upper limit of their 5 (and 0) in their integrated function.
This may appear as two separate integrals 0 to 3 then 3 to 5

B1 Area of rectangle = 90 or 18×5
.

M1 Uses area of rectangle $-$ area under curve (either way around). It is dependent upon both previous M's

A1 cso $57\frac{7}{24}$ Note $-57\frac{7}{24}$ is A0

……

Method two: Curve - line or line - curve

M1 For integrating what they think is their $\pm(\text{'18'}-\text{f}(x))$ which must be cubic.
All powers must be raised by one for this to be scored.

A1 Correct $\pm\left(-\frac{1}{8}x^4+\frac{1}{3}x^3+\frac{15}{4}x^2\right)$ which may be unsimplified

M1 Uses an upper limit of their 5 (and 0) in their integrated function.
This may appear as two separate integrals 0 to 3 then 3 to 5

B1 Area of rectangle implied by $\pm57\frac{7}{24}$ **There is no need to use a calculator on incorrect functions (score B0)**

M1 Uses area of rectangle $-$ area under curve (either way around). It is dependent upon both previous M's
Can be awarded on line 1

A1 cso $57\frac{7}{24}$ Note $-57\frac{7}{24}$ is A0

……

Special case: There will be quite a few candidates who believe that the equation is $y=x^3-2x^2-15x+36$

(d)	**Method 1** $\int\left(x^3-2x^2-15x+36\right)\mathrm{d}x=\frac{1}{4}x^4-\frac{2}{3}x^3-\frac{15}{2}x^2+36x$	**Method 2** OR $\int\left(-x^3+2x^2+15x-18\right)\mathrm{d}x=-\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{15}{2}x^2-18x$	M1A0
	Uses their 5 as the upper limit (and subtracts 0) to obtain an area	Uses their 5 as the upper limit (and subtracts 0) to obtain area	M1
	Area of rectangle = 90	Implied by answer $\pm 24\frac{7}{12}$	B1
	Use = Area of rectangle – Area beneath curve	Implied by subtraction in the integration	M1
	$=90-65\frac{10}{24}=24\frac{7}{12}$	$=24\frac{7}{12}$	A0
			4/6

……….

For answers without working which seem to be quite common

Eg. Area $=90-\int_0^5\frac{(x-3)^2(x+4)}{2}\mathrm{d}x=\frac{1375}{24}$ score M0 A0 M1 (limits) B1 (90) M0 (Both M's needed) A0 for 2/6

………..

Handy Marking guide for 15 d,

Ist M1	For integrating a cubic that resulted from Multiplying out $\frac{(x-3)^2(x+4)}{2}$, $(x-3)^2(x+4)$ or solving $\frac{(x-3)^2(x+4)}{2}="18"$ Don't worry if there are errors. Score for a cubic going to a quartic with all powers being raised by one
A1	Can only be scored for: Method one $\frac{1}{8}x^4-\frac{1}{3}x^3-\frac{15}{4}x^2+18x$ Method two $\pm\left(-\frac{1}{8}x^4+\frac{1}{3}x^3+\frac{15}{4}x^2\right)$
M1	Uses their 5 as the upper limit (and subtracts 0) to obtain an area
B1	Method One for sight of 90 or 18×5 Method Two for a (correct) answer of $\pm57\frac{7}{24}=\frac{1375}{24}$ or $\pm24\frac{7}{12}=\pm\frac{295}{12}$ in the special case
dM1	It is dependent upon both previous M's Method One Rectangle - area under curve Method Two Awarded on line 1 for integral (curve-18) either way around
A1	Cso $57\frac{7}{24}$ or $\frac{1375}{24}$

Question Number	Scheme	Marks
16.	$\frac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+2$	B1
	Sub $x=1, y=4 \Rightarrow y=ax^3+bx^2+2x-5$ or $x=1$ into $ax^3+bx^2+2x-5=12x-8$	M1
	Sub $x=1, \frac{\mathrm{d}y}{\mathrm{d}x}=12 \Rightarrow 3a+2b+2=12$	M1
	Solves simultaneously $a+b=7, 3a+2b=10 \Rightarrow a=-4, b=11$	dM1A1
		[5]
		(5 marks)

B1 States or uses $\frac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+2$

M1 Attempts to substitute $x=1, y=4$ in $y=\mathrm{f}(x) \Rightarrow a+b+2-5=4$

This also can be scored by to substituting $x=1$ into $ax^3+bx^2+2x-5=12x-8 \Rightarrow a+b+2-5=12-8$

M1 Attempts to substitute $x=1, \frac{\mathrm{d}y}{\mathrm{d}x}=12$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+2$

dM1 Solves simultaneously to find both *a* and *b*. Both M's must have been awarded. Allow from a graphical calculator. Sight of both values is sufficient.

A1 $a=-4, b=11$

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